Finitely non-standard models of Robinson arithmetic

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To the best of my knowledge, there is no systematic study of finitely non-standard models of Robinson arithmetic[1]---that is, there is no systematic study of those models of Robinson arithmetic that have a non-empty finite set of non-standard numbers. I have made a modest attempt at such a systematic study (Lundstedt, 2025).

The main results are that for each such finitely non-standard model:

- The successor function is a permutation of the set of non-standard numbers (this follows trivially from the axioms). As is well-known, each permutation of a finite set has a unique decomposition into cycles on disjoint orbits.

- The restriction of addition to

  {⟨a,n⟩ : a non-standard, n standard}

  is “tame”, in the sense that this restriction is determined already by the permutation given by the successor function:

  a+n =
  the result of starting at a and taking n steps in its cycle.

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[1] I work with Robinson's (1950) original axiomatization:

$$
\begin{aligned}
Sx &\neq 0 \\
Sx &= Sy \rightarrow x = y \\
x &= 0 \lor \exists y\colon x = Sy \\
x+0 &= x \\
x+Sy &= S(x+y) \\
x\times 0 &= 0 \\
x\times Sy &= x\times y+x.
\end{aligned}
$$

- The restriction of addition to

  $\{\langle\alpha,a\rangle : \alpha$ standard or non-standard, a non-standard$\}$

  is “wild”. Contrary to the previous restriction, this one is not determined by the successor function, but subject to some constraints---in particular, its range may only consist of non-standard numbers.

- Similar to the case of addition, the restriction of multiplication to

  $\{\langle a,n\rangle : a$ non-standard, n standard$\}$

  is determined by the successor function and addition, whereas the restriction of multiplication to

  $\{\langle\alpha,a\rangle : \{\alpha$ standard or non-standard, a non-standard$\}$

  is not uniquely determined (by the successor function and addition), but subject to some constraints.

REFERENCES

- Lundstedt, Anders E.V. (2025): Finitely non-standard models of Robinson arithmetic; version 2025-06-10. In: Natural induction (a model-theoretic study); PhD thesis in preparation. URL:

  https://anderslundstedt.com/research/#induction-drafts

- Robinson, Raphael M. (1950): An essentially undecidable axiom system. In: Proceedings of the International Congress of Mathematicians; volume I, pages 729--730; American Mathematical Society; Providence 1952.