▸ ERIC JOHANNESSON, ANDERS LUNDSTEDT, *When one must strengthen one's induction hypothesis.*
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Sometimes when trying to prove a fact by induction, one gets "stuck" at the induction step. The solution is often to use a "stronger" induction hypothesis, that is to prove a "stronger" result by induction. But in such cases, can we say that "strengthening the induction hypothesis" is necessary in order to prove the fact?

The general problem of when one must, in order to prove a fact X, first prove another fact Y, seems very hard. Interestingly, the special case of when one must strengthen one's induction hypothesis turns out to be more manageable. We provide the following characterization of when one in fact must strengthen one's induction hypothesis.

Let $\mathsf{Th}(\mathcal{N})$ be the set of sentences of first-order arithmetic that are true in the standard model. Let $T \subseteq \mathsf{Th}(\mathcal{N})$ and let $\varphi(x)$ and $\psi(x)$ be formulas both with at most one free variable x. Say that $\psi(x)$ *witnesses that* T *proves* $\forall x \varphi(x)$ *with and only with strengthened induction hypothesis* if and only if

(1) $T \cup \{\varphi(0) \wedge \forall x(\varphi(x) \to \varphi(x+1)) \to \forall x \varphi(x)\} \nvdash \forall x \varphi(x)$,
(2) $T \vdash \varphi(0)$,
(3) $T \vdash \psi(0)$,
(4) $T \vdash \forall x(\psi(x) \to \psi(x+1))$,
(5) $T \vdash \forall x \psi(x) \to \forall x \varphi(x)$.

We show that this definition applies to a number of natural examples. By reflecting on mathematical practice, we argue that this definition does capture the notion of "proof by strengthened induction hypothesis".